ANOVA
(Analysis of Variance)

Dr. Baxi

Introduction

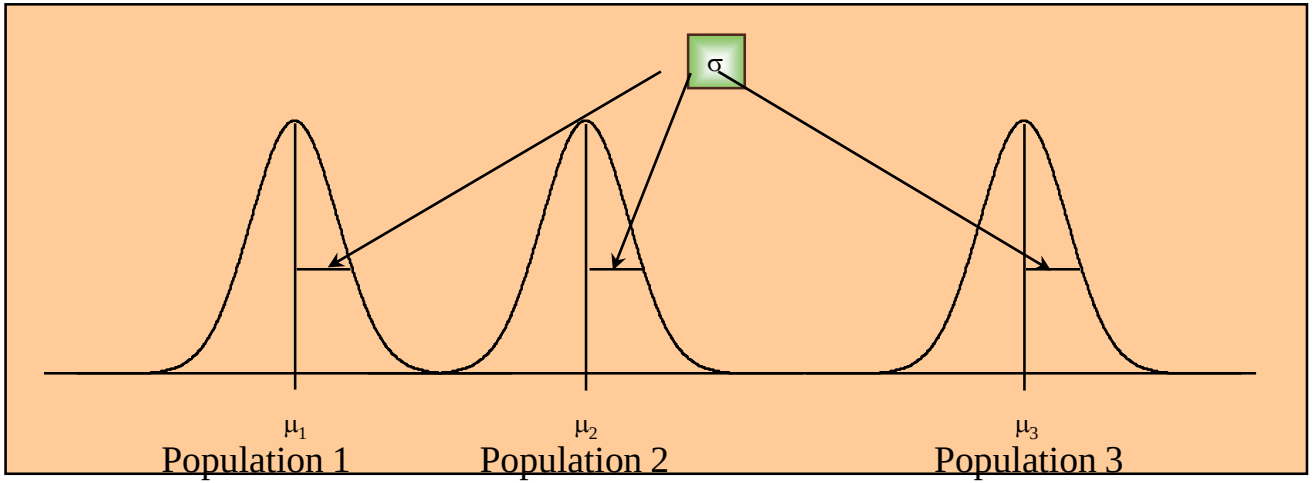
- ANOVA is an abbreviation for the full name of the method: Analysis Of Variance.
- ANOVA is used to test the significance of the difference between more than two sample means and to make inferences about whether our samples are drawn from population having same means.
- ANOVA is comparison of means. Each possible value of a factor or combination of factor is a treatment.
- The ANOVA is a powerful and common statistical procedure in the social sciences. It can handle a variety of situations.

Why ANOVA instead of multiple t-tests?

- If you are comparing means between more than two groups, why not just do several two sample t-tests to compare the mean from one group with the mean from each of the other groups?
- Before ANOVA, this was the only option available to compare means between more than two groups.
- The problem with the multiple t-tests approach is that as the number of groups increases, the number of two sample t-tests also increases.
- As the number of tests increases the probability of making a Type I error also increases.

Analysis of Variance : Assumptions

- We assume *independent random sampling* from each of the *r* populations
- We assume that the *r* populations under study:
  - are *normally distributed*,
  - with means μ_i that may or may not be equal,
  - but with *equal variances*, σ_i^2.



ANOVA Hypotheses

- The Null hypothesis for ANOVA is that the means for all groups are equal:

$$H_o : \mu_1 = \mu_2 = \mu_3 = ... = \mu_k$$

- The Alternative hypothesis for ANOVA is that *at least two* of the means are not equal.

- The test statistic for ANOVA is the ANOVA F-statistic.

SST = SSW + SSB

- This partitioned relationship is also true for the squared differences:
  - The variability between each observation and the overall (or grand) mean is measured by the 'sum of squares total' (SST)
  - The variability within groups is measured by the 'sum of squares within' (SSW).
    - MSW = SSW/(n-k)
  - The variability between groups is measured by the 'sum of squares between' (SSB).
    - MSB = SSB/(k-1)

One & N way ANOVA

- One way ANOVA

  Analysis of variance, so named because they can consider only one independent variable at a time.

- N way ANOVA

  As its name suggests, this is a procedure that allows you to examine the effects of n independent variables concurrently.

ANOVA example: mobility

The hypothetical data below represent mobility scores (higher score indicates improved mobility) for 3 groups of patients:

Control group did not receive any therapy

Treatment group 1 received physical therapy,

Treatment group 2 received counseling and physical therapy.

Assume that the mobility scores are normally distributed.

Control	Trt. 1	Trt. 2
35	38	47
38	43	53
42	45	42
34	52	45
28	40	46
39	46	37

Example continued..

- State the Hypotheses
  - Null Hypothesis: $m_{control} = m_{trt\ 1} = m_{trt\ 2}$
  - Alternative Hypothesis: at least two of the means ($m_{control}$, $m_{trt\ 1}$, $m_{trt\ 2}$) are not equal
  - ANOVA is always a two-sided test
  - ANOVA will identify if at least two means are significantly different but will not identify **which two (or more) means are different.**
- Set the significance level
  - a = 0.05.

Example continued..

- Calculated SD for each group
  - Control: 4.86
  - Trt 1: 4.94
  - Trt 2: 5.33
- Calculated ratio:
  - largest SD / smallest SD = 5.33 / 4.86 = 1.1
- Since the ratio < 2, assume equality of variance between groups

Example continued..

- Overall mean = average of all 18 observations = 41.7
- Group means = average of the observations in each group
  - Control mean = 36
  - Treatment 1 mean = 44
  - Treatment 2 mean = 45

Calculate the Within Sum of Squares: SSW

- Square the difference between each observation and it's group mean and sum the 18 terms.
- The SSW for the control group (mean = 36)

  $(35 - 36)^2 + (38 - 36)^2 + (42 - 36)^2 + (34 - 36)^2 + (28 - 36)^2 + (39 - 36)^2 = 118$
- The SSW for the treatment 1 group (mean = 44)

  $(38 - 44)^2 + (43 - 44)^2 + (45 - 44)^2 + (52 - 44)^2 + (40 - 44)^2 + (46 - 44)^2 = 122$
- The SSW for the treatment 2 group (mean = 45)

  $(47 - 45)^2 + (53 - 45)^2 + (42 - 45)^2 + (45 - 45)^2 + (46 - 45)^2 + (37 - 45)^2 = 142$
- SSW = sum of SSW for each group = 118 + 122 + 142 = 382
- Now MSW = SSW/N-K so, 382/15 = 25.47

Calculate the Between Sum of Squares: SSB

- The overall mean = 41.7
- The three group means are:
  - Control: mean = 36
  - Treatment 1: mean = 44
  - Treatment 2: mean = 45
- For each group square the difference between the group mean and the overall mean and multiply by the group sample size, then sum these 3 terms for the SSB:

$SSB = 6*(36 - 41.7)^2 + 6*(44 - 41.7)^2 + 6*(45 - 41.7)^2 = 292$

MSB = SSB/K-1 so MSB = 292/2

MSB = 146

Calculate the ANOVA F-statistic

- The ANOVA F-statistic = MSB/MSW
- The ANOVA F-statistic will be large when there is more variability *between* the groups than *within* the groups.
- If the variability between groups and within groups is approximately equal the ANOVA F-statistic will be small (close to 1.0)
- The Null hypothesis of equal means between groups is rejected if the F-statistic is large enough.
- ANOVA F-statistic for example = 146/25.47 = 5.73

Find the p-value of the ANOVA F-statistic

- The p-value of the ANOVA F-statistic is the right tail area greater than the F-statistic under the

  F-distribution with (numerator distribution frequency, denominator distribution frequency)
- The F-statistic for the example data = 5.73
- The df for the F-distribution are (3-1) = 2 for the numerator and (18-3) = 15 for the denominator.
- p value of 5.73, 2, 15 = 0.014
- Since the p-value of 0.014 (3.68) < the significance level of 0.05, the null hypothesis of equality between all three group means is rejected.
- We can conclude that that AT LEAST two of the means are significantly different.

Application of ANOVA

- ANOVA is designed to detect differences among means from populations subject to different treatments.
- ANOVA is a joint test

  The equality of several population means is tested simultaneously or jointly.
- ANOVA tests for the equality of several population means by looking at two estimators of the population variance (hence, analysis of variance).
- Product testing, ad copy testing and concept testing are some common applications, though ANOVA Analysis Surveys can be used in retail environments or simulated lab-type environments

Thank You