Deterministic Operations Research Models

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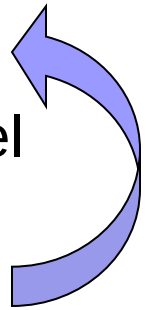
What is Operations Research?

- Enterprises typically consist of various units whose operations need to be coordinated
  - A typical problem: how to distribute resources across units so as to maximize efficiency
- OR is the discipline of applying analytical tools based on mathematical models to help take better decisions in managing these operations (the "science of better")
- Typical decision-making process in OR entails:
  1. Gathering available data
  2. Building an abstract mathematical model
  3. Solving the mathematical problem
  4. Supplying the results to management



Feedback: may need to enhance data or model

A Brief History of OR

World War II led to the birth of OR

- Scientists and engineers used mathem
  deploy, manage, and analyze military op
  - deployment of the radar
  - management of convoy and submarine

Perhaps the single most important catalyst in the advancement of OR

- Many important developments took place in this period. Most notably George Dantzig invented the simplex method for solving linear programs (LPs) in 1947
  - One of the first applications was the diet problem: given foods with varying nutrient amounts, plan a diet that satisfies the desired nutrient requirements + the food-amount constraints, and minimizes cost

Brief History (contd.)

After the War:

- Industrial boom led to rapid increase in size of corporations – growing need for systematic decision-making tools
- Managers began to realize both, the modeling power of LPs and OR tools, and their potency in improving efficiency even under existing technology; applications increased manifold
- Serendipitously, great advances were made in computational technology, which allowed one to solve problems of ever-increasing size via OR tools

INTRODUCTION & HISTORY OF O.R.

- It is generally agreed that O.R. came into the existence as a discipline during the world war-2.
- The term of O.R. was coined as a result of research on military operations during this war.
- Since the war involved strategic and tactical problems which were so complicated.
- That to expecting adequate solution from individual or specialists in a single discipline was unrealistic. to be con…

INTRODUCTION & HISTORY OF O.R.

- Therefore, the group of individual who collectively were considered specialists in mathematics, economics, statistics and probability theory, engineering, behavioral and physical science were formed as a special units within the armed forces to deal with strategic and tactical problems of various military operation.
  To be con….

INTRODUCTION & HISTORY OF O.R.

- A key person in post-war development of O.R. was George B. Dantzig.
- In India, Operations Research came into existence in 1949 when An O.R. unit was established at Regional Research Laboratory , Hyderabad. At the same time Prof. R. S. Verma, also set up an O.R. team at Defense Science Laboratory to solve problems of store, purchase and planning. To be con…

Definitions Of O.R.

- Because of the wide scope of the applications of operations research, giving a precise definition is difficult. However, a few definitions of O.R. are as under:

Definitions Of O.R.

- O.R. is concerned with scientifically deciding how to best design and operate man-machine system usually requiring the allocation of scarce resource.

  (Operations Research Society, America.

- O.R. is the systematic application of quantitative methods, techniques and tools to the analysis of problems involving the operation of system .

  (Daellenbach and George, 1978)

Definitions Of O.R.

- O.R. is the art of giving bad answers to problems to which otherwise worse answers are given. (T.L.Saaty,1958)
- O.R. may be described as a scientific approach to decision making that involves the operation of organizational system.

(F.S.Hiller and G.J.Lieberman ,1980)

Definitions Of O.R.

- Operations Research is a scientific approach to problems solving for executive management.

H. M.Wanger.

CHARACTERISTICS OF O.R.

- O.R. uses mixed team approach to find out optimum solution.
- O.R. uses scientific method to arrive at the optimum solution.
- O.R. is the inter disciplinary team approach to find out the optimum return.
- O.R. emphasis on the overall approach to the system. i.e. All the aspects of the problem under consideration.

CHARACTERISTICS OF O.R.

- O.R. tries to optimize the total output by maximizing the profit and minimizing the loss or cost.
- O.R. gives bad answer to the problems where worse could be given. i.e. it cannot give perfect answers to the problem thus or improves only the quality of the solution.

CHARACTERISTICS OF O.R.

- The task of operation researcher may be said to consist of studying an operation selecting variables to described the operation ; investigating the mathematical relationships of the variables.
- Operation Researcher actually determines what factor influence the state of affairs and tries to measure them. All these measurements are then incorporated in to a generalized system.
- O.R. deals more in experiments and induction rather than in analysis and deduction.

PHASES OF SCIENTIFIC METHOD IN O.R. APPROACH

- **GUDGEMENT PHASE**
- **RESEARCH PHASE**
- **ACTION PHASE**

JUDGEMENT PHASE

- This phase includes
- Identification of the real-life problem ,
- Selection of an appropriate objective and the values of various variables related to this objective,
- Application of the appropriate scale of measurement , i.e. deciding the measures of effectiveness, and

JUDGEMENT PHASE

- Formulation of an appropriate model of the problem, abstracting the essential information so that a solution at the decision-maker’s goals can be sought.

RESEARCH PHASE

- This phase is the largest and longest among other two phases. However, the remaining two are also equally important as they provide the basis for a scientific method. This phase utilizes:
- Observations and data collection for a better understanding of the problem,
- Formulation of hypothesis and models,
- Observation and experimentation to the test hypothesis on the basis of the additional data,

RESEARCH PHASE

- Analysis of the available information and verification of the hypothesis using pre-established measures desirability,
- Predictions of various results from the hypothesis, and
- Generalization of the result and consideration of alternative methods.

ACTION PHASE

- This phase consists of making recommendation for implementing the decision by an individual who is in the position tom implement results. This man must be aware of the environment in which the problem occurred, objective, assumption, and omission of the model of the problem.

OR in action: Optimizing Harbor Operations

Picture of container terminal Altenwerder in Hamburg

One of the most modern terminals: handles ~ 2.4 million containers yearly

All internal traffic and storage cranes are automated.

Where is OR Today?

- Immense computing power available readily and fairly cheaply, e.g., PCs
- A half-century of research in OR has led to:
  - very good theoretical understanding
  - various software packages, e.g., CPLEX, LINDO, XPRESS-MP, being available that can be used "off-the shelf"
- OR is everywhere – from booking an airline ticket, to checking into your hotel
- By the end of this course, you will be able to solve real-world problems on your PC using these OR tools!

Harbor Operations (contd.)

- Overall objective: minimize vessel waiting time
- Containers are transported between vessels and storage area via automated guided vehicles (AGVs)

Moehring et al. 2004 give a fast algorithm based on shortest-path computations that works much better than previous adhoc methods

Subproblem: compute AGV routes that are free of conflicts and jams while maximizing container throughput.
Also routes have to be computed in real-time.

Some other success stories

- Continental Airlines saves $40 million by using OR tools to near-optimally reassign crews after disruptions
- Texas Children's Hospital used nonlinear optimization to monitor healthcare contract negotiations
- Athens Olympic Organizing Committee used logistics, optimization tools to manage its resources and plan the 2004 Olympics; estimated savings: $70 million
- Philips Semiconductors saves $5 million by using stochastic multiperiod inventory theory to handle demand uncertainty
- Many more applications on "The Science of Better" website on Syllabus page

Yet, it is always surprising (to me) that there are many applications out there that are tailor-made for the use of OR tools that are still tackled by adhoc methods.

Article in ScienceDaily titled

"Techniques for making Barbie Dolls can Improve Health Care"

talks about how OR tools will soon come to be adopted to improve health care delivery

Researchers working on optimizing Air New Zealand's crew scheduling reported that "many airlines still use heuristic or manual methods"

What you will learn

- Mathematical Modeling
  - learn a variety of ways of modeling real-world problems as structured mathematical problems
- Solution Methods
  - learn to use powerful optimization tools to solve the problems arising in your mathematical models

⇉MODELS & MODELLING IN O.R.

- Models attempt to describe the essence of a situation or activity by abstracting from reality so the decision-maker can study the relationship among relevant variables in isolation. Hence, models do not, can not, represent every aspect of reality because of the innumerable and changing characteristics of the real life problems to be represented.
- From the above discussion, it appears that a model is constructed to analyze and understand the given system for the purpose of improving its performance.
- There are many ways to classify models. Classification schemes can also provide a useful frame of reference for modelers. There are eight classification schemes for models.

Model classification schemes

1) Function
   - Descriptive
   - Predictive
   - Normative
2) Structure
   - Iconic
   - Analog
   - Symbolic
3) Dimensionality
   - Two-dimensional
   - Multi-dimensional
4) Degree of certainty
   - Certainty
   - Conflict
   - Risk
   - uncertainty
5) Time reference
   - Static
   - Dynamic
6) Degree of generality
   - Specialized
   - General
7) Degree of closure
   - Closed
   - Open
8) Degree of quantification
   - Qualitative
   - Mental
   - Verbal
   - Quantitative
   - Statistical
   - Heuristic
   - Simulation

⇉Classification based on structure

- ***Physical models***

These models are useful only in design problems because they are easy to observe, build and describe. (e.g. in the aircraft industry, scale models of the proposed new aircraft are built and tested in wind tunnels to record the stresses experienced by the air frame. These models are not useful for prediction and can't be analysed with a physical model.

1.Iconic Models- Iconic models retain some of the physical properties and characteristics of the system they represent. (e.g. blueprints of a home, maps,globes,photographs,drawings,airplanes,trains,etc.)

2.Analogue Models- these models represent a system by a set of properties different from those of the original system and doesn't resemble it physically. After the problem is solved, the solution is re-interpreted in terms of the original system. (e.g. the organizational chart represents the state of formal relationships existing between members of the organization.)

- ***Symbolic models***

  these models use symbols (letters, numbers) and functions to represent variables and their relationships to describe the properties of the system. These models can be classified into two categories.

  1. verbal models- these models describe a situation in written or spoken language. (e.g. written sentences, books, etc.)

  2. mathematical models- these models involve the use of mathematical symbols, letters, numbers and mathematical operators (+, -,..) to represent relationships among various variables of the system to describe its properties or behaviour.

Classification based on Function or Purpose

- Models based on the purpose of their utility include the following types.

- ***Descriptive models*** –

Descriptive models simply describe some aspects of a situation, based on observation, survey, questionnaire results or other available data of a situation and don't predict or recommend anything. (e.g. organization chart, plant layout diagram, block diagram representing an algorithm or method for solving a problem, etc.)

- ***Predictive models*** –

These models are used to predict the outcomes due to a given set of alternatives for the problems. These models don't have an objective function as a part of the model to evaluate decision alternatives. (e.g. S = a + bA+ cI)

- ***Normative (or Optimization) models*** –

These models provide the 'best' or 'optimal' solution to problems subject to certain limitations on the use of resources. These models are also called "prescriptive models" because they prescribe what the decision-maker ought to do.

→ Classification based on the time reference

- ***Static models*** –

  These models represent a system at some specified time and don't account for changes over time.

- ***Dynamic models*** –

  In a dynamic model, time is considered as one of the variables and allows the impact of changes due to change in time. Dynamic programming is an example of a dynamic model.

→ Classification based on degree of certainty

- ***Deterministic models***-

  if all the parameters, constants and functional relationships are assumed to be known with certainty when the decision is made, the model is said to be deterministic.

- ***Probabilistic (Stochastic) models***-

  models in which at least one parameter or decision variable is a random variable are called probabilistic (or stochastic) models.

  insurance against risk of fire, accidents, sickness, etc. are examples where the pattern of events is studied in the form of a probability distribution.

→ Classification based on method of solution or quantification

- ***Heuristic models***-

  These models employ some sets of rules which, though perhaps not optimal, do facilitate solutions of problems when applied in a consistent manner.

- ***Analytical models***-

  these models have a specific mathematical structure and thus can be solved by known analytical or mathematical techniques. Any optimization model (which requires maximization or minimization of an objective function) is an analytical model.

- ***Simulation models***-

  These models also have a mathematical structure but are not solved by applying mathematical techniques to get a solution. These models don’t provide general solution like those of mathematical models.

⇉ General methods for solving O.R. models

- ***Analytical (or Deductive) method***-

  In this method, classical optimization techniques such calculus, finite difference and graphs are used for solving an O.R. model.

  TC = (D/Q) Co + (Q/2) Ch ,

  where TC= total variable inventory cost,

  Co= ordering cost per order,

  D= annual demand,

  Q= size of an order,

  Ch= carrying cost per time period.

- ***Numerical (Iterative)method***-

  In this method, instead of solving the problem directly, a general algorithm is applied to obtain a specific numerical solution. This model starts with a solution obtained by trial and error, and a set of rules for improving it towards optimality.

- ***Monte Carlo method***-

  This method is based upon the idea of experimenting on a mathematical model by inserting into the model specific values of decision variables at different points of time and under different conditions and then observing their effect on the criterion chosen for variables.

Advantages of models

1) A model provides economy in representation of the realities of the system.

2) The problem can be viewed in its entirety, with all the components being considered simultaneously.

3) Models serves as aids to transmit ideas and visualization among people in the organization.

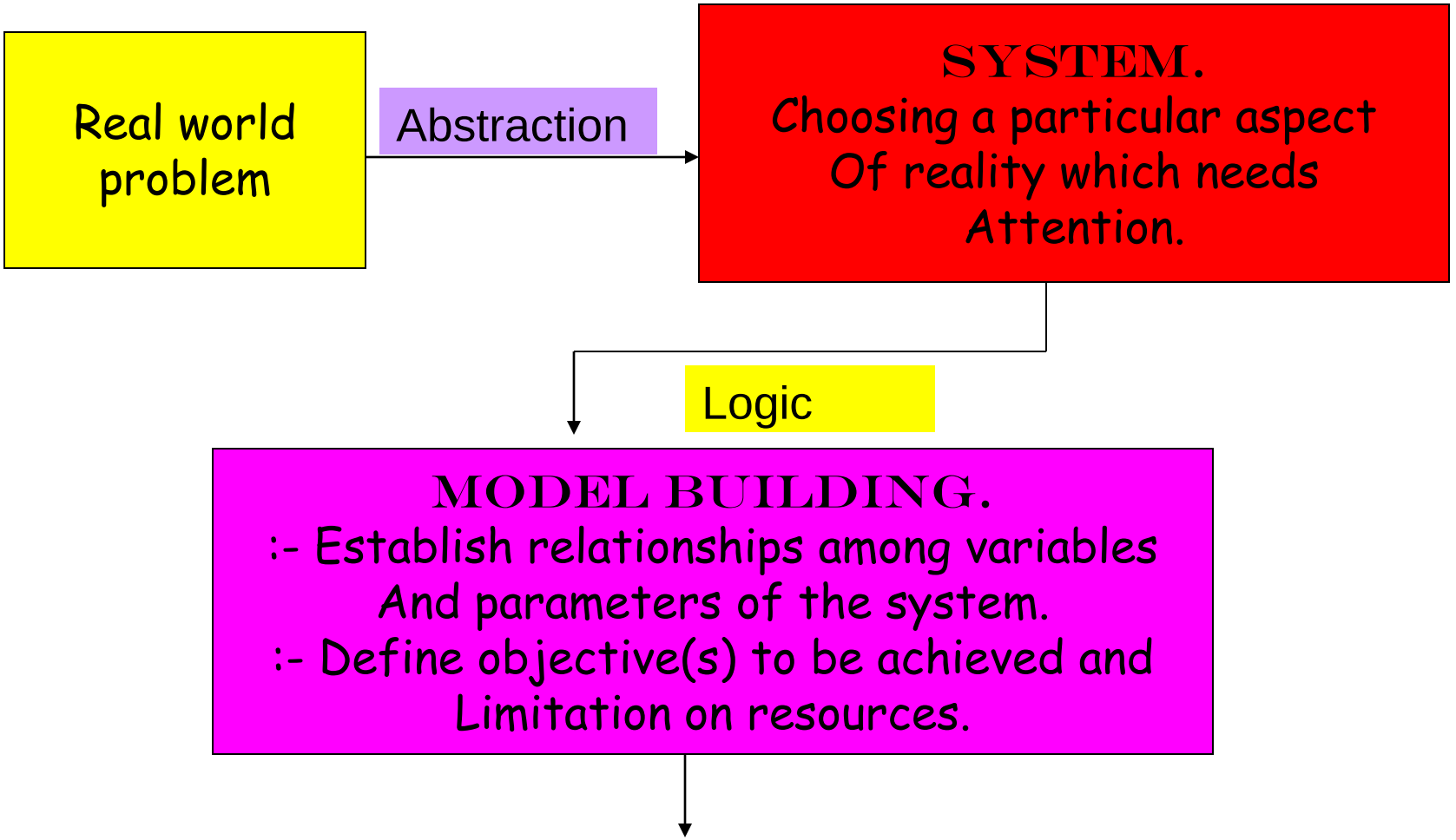
4) A model allows us to analyse and experiment in a complex situation to a degree that would be impossible in the actual system and its environment.

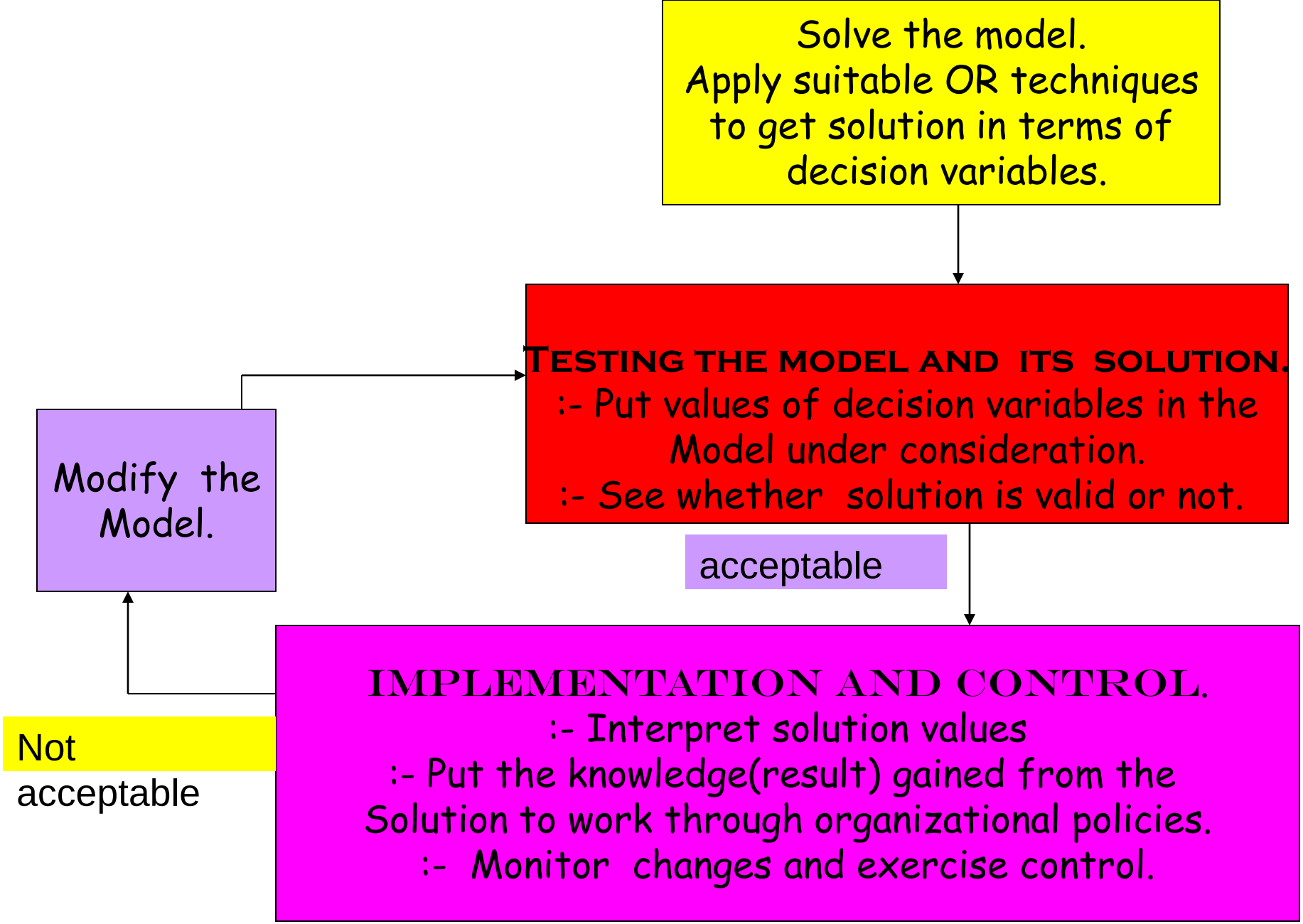
5) Models simplify the investigation considerably and provide a powerful and flexible tool for predicting the future state of the process or system.

METHODOLOGY OF OPERATIONS RESEARCH

Every OR specialist may have his\her own way of solving problems. However,for effective use of OR techniques, it is essential to follow some steps that are helpful for decision-maker to make better decisions.The flow diagram representating the methodology of OR is also shown in Fig. 1.1

FIG. 1.1


Real world problem
Abstraction
SYSTEM.
Choosing a particular aspect
Of reality which needs
Attention.
Logic
MODEL BUILDING.
:- Establish relationships among variables
And parameters of the system.
:- Define objective(s) to be achieved and
Limitation on resources.


Solve the model.
Apply suitable OR techniques to get solution in terms of decision variables.
TESTING THE MODEL AND ITS SOLUTION.
:- Put values of decision variables in the Model under consideration.
:- See whether solution is valid or not.
acceptable
IMPLEMENTATION AND CONTROL.
:- Interpret solution values
:- Put the knowledge(result) gained from the Solution to work through organizational policies.
:- Monitor changes and exercise control.
Not acceptable
Modify the Model.

Step 1 Formulating the problem

Problem formulation involves an analysis of the system under study, the objectives of the decision-maker, and alternative courses of action, etc.., as to understand and describe, in precise terms, the problem that an organization faces.

The major steps which have to be taken into consideration for formulating the problems are as follows:

- problem components
- Decision environment
- Alternative courses of action
- Measures of effectiveness

Step 2 collecting data and constructing a mathematical model

Certain basic components required in every decision problem model are :

- Decision variables
- Uncontrollable variables
- Objective function
- Constraints
- Functional relationships
- parameters

Step 3 Solving the mathematical model

In general, the following two methods are used for solving an OR model

- optimization methods
- heuristic methods

Step 4 Validating the solution

Step 5 Implementing the solution

Step 6 Modifying the model

Step 7 Establishing Controls over the solution

Advantages of OR study

- A decision-maker in business and industry comes across the problem of making decisions in his day to day activities.

- In addition to learning about various OR models useful in making a decision, the decision-maker can also get several other advantages listed below.

1. Structured approach to problems
2. Critical Approach to problem solving

Opportunities and shortcoming of OR

« **Opportunities**

1. It makes the decision-maker consider very carefully just what variables influence the decisions.

3 Quickly points out gaps in data required to support workable solution to problem.

1. It compels the the decision-maker -maker to be quite explicit about his objective, assumptions and his perspective to constraints.

4 it models can be solved by a computer, thus a management can get through time for decisions that require quantitative approach.

Shortcomings

- Often solution to a problem is derived either by making it simplified or simplifying assumptions and thus, such solutions have limitations
- Sometimes models do not represent the realistic situations in which decisions must be made .
- Often decision-maker is not fully aware of the limitation of the models that he is using.
- Many real world problems just cannot have an OR solution.

Features of operations research solution

- Technologically appropriate : The solution should work technically, meet the constraints and operate in the problem environment.
- Reliable
- economically viable
- Behaviourally appropriate

Applications and scope of OR

Finance and Accounting

Marketing

Purchasing, procurement and exploration

Facilities planning

Manufacturing

Maintenance and project scheduling

Personnel management

Techniques and General management

Government

Basic OR models

- Allocation models
- Inventory models
- Waiting lines models
- Competitive models
- Network Models
- Sequencing models
- Replacement models
- Dynamic Programming models
- Markov-chain models
- Simulation models
- Decision Analysis models